The position of an object at time t is given by $s(t) = (t^3 - 5t)3^t$. **SCORE: _____ / 5 PTS**

Find the **acceleration** of the object at time $t = 2$.

$$s'(t) = (t^3-5t)'3^t + (t^3-5t)(3^t)' = \boxed{(3t^2-5)3^t} + \boxed{(t^3-5t)3^t \ln 3}$$

1 1

$$s''(t) = (3t^2-5)'3^t + (3t^2-5)(3^t)' + (t^3-5t)'3^t\ln 3 + (t^3-5t)(3^t \ln 3)'$$

$$= \boxed{6t \cdot 3^t} + \boxed{(3t^2-5)(3^t \ln 3)} + \boxed{(3t^2-5)(3^t \ln 3)} + \boxed{(t^3-5t)(3^t(\ln 3)^2)}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$$s''(2) = 12 \cdot 9 + 7 \cdot 9 \ln 3 + 7 \cdot 9 \ln 3 + (-2) \cdot 9(\ln 3)^2$$

$$= \boxed{108 + 126 \ln 3 - 18(\ln 3)^2}$$ 1

Prove the derivative of $\sec x$ using the definition of the derivative function. **Show all steps.** **SCORE: _____ / 5 PTS**

Do NOT use the quotient rule, nor the known derivatives of any other trigonometric functions.

You may use the value of the two limits proved in lecture without proving them again.

$$\lim_{h\to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h} \quad \frac{1}{2}$$

$$= \lim_{h\to 0} \frac{\cos x - \cos(x+h)}{h\cos(x+h)\cos x} \quad \frac{1}{2}$$

$$= \lim_{h\to 0} \frac{\cos x - (\cos x \cos h - \sin x \sin h)}{h\cos(x+h)\cos x} \quad 1$$

$$= \lim_{h\to 0} \frac{\cos x(1-\cos h) + \sin x \sin h}{h\cos(x+h)\cos x}$$

$$= \lim_{h\to 0} -\frac{1}{\cos(x+h)} \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} \quad \frac{1}{2}$$

$$+ \lim_{h\to 0} \frac{\sin x}{\cos(x+h)\cos x} \cdot \lim_{h\to 0} \frac{\sin h}{h} \quad \frac{1}{2}$$

$$= -\frac{1}{\cos x} \cdot 0 + \frac{\sin x}{\cos^2 x} \cdot 1 \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sec x \tan x \quad \frac{1}{2}$$

Find the slope of the tangent line to the curve $y = \dfrac{f(x)}{x^2}$ at the point where $x = 3$ if $f(3) = -4$ and $f'(3) = 2$. **SCORE: _____ / 4 PTS**

$$\frac{dy}{dx} = \frac{f'(x)\,x^2 - f(x)(x^2)'}{(x^2)^2} = \boxed{\frac{x^2 f'(x) - 2x\,f(x)}{x^4}}$$

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$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{3^2 \cdot f'(3) - 2(3) \cdot f(3)}{3^4} = \boxed{\frac{9 \cdot 2 - 6 \cdot (-4)}{81}} = \frac{42}{81} = \boxed{\frac{14}{27}}$$

$1\frac{1}{2}$ $\qquad$ $\frac{1}{2}$

Prove the derivative of $\cot x$ using the quotient rule. **Show all steps.** SCORE: _____ / 3 PTS

You may use the known derivatives of *sin x* and *cos x* without proving them.

$$\cot x = \frac{\cos x}{\sin x}$$

$$(\cot x)' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{(\sin x)^2}$$

$$= \frac{\boxed{(-\sin x)\sin x - \cos x(\cos x)}\ 1\tfrac{1}{2}}{\boxed{(\sin x)^2}\ \tfrac{1}{2}}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \boxed{-\frac{1}{\sin^2 x}}\ \tfrac{1}{2}$$

$$= \boxed{-\csc^2 x}\ \tfrac{1}{2}$$

$$\frac{d}{dx}\frac{2-3x^2+x^4}{1-2x^4}$$ (Your final answer must be one fraction)

$$=\frac{\boxed{(-6x+4x^3)(1-2x^4)-(2-3x^2+x^4)(-8x^3)}}{\boxed{(1-2x^4)^2}} \quad 2\tfrac{1}{2} \quad \tfrac{1}{2}$$

$$=\frac{-6x+12x^5+4x^3\cancel{-8x^7}+16x^3-24x^5\cancel{+8x^7}}{(1-2x^4)^2}$$

$$=\boxed{\frac{-12x^5+20x^3-6x}{(1-2x^4)^2}} \quad 1$$

$$\frac{d}{dy}(5y^e + 7e^y - 3\pi^e)$$

$= \underbrace{5ey^{e-1}}_{1} + \underbrace{7e^y}_{1}$

$+\frac{1}{2}$ FOR NOT WRITING A DERIVATIVE FOR $-3\pi^e$ (IE. DERIVATIVE $= 0$)

$$\frac{d^3}{dr^3}\frac{18r^2-27r}{4\sqrt[3]{r}}$$

$$= \frac{d^3}{dr^3}\left(\frac{9}{2}r^{\frac{5}{3}} - \frac{27}{4}r^{\frac{2}{3}}\right)$$

$$= \frac{d^2}{dr^2}\left(\boxed{\frac{15}{2}r^{\frac{2}{3}} - \frac{9}{2}r^{-\frac{1}{3}}}\right) \quad 1\tfrac{1}{2}$$

$$= \frac{d}{dr}\left(\boxed{5r^{-\frac{1}{3}} + \frac{3}{2}r^{-\frac{4}{3}}}\right) \quad 1$$

$$= \boxed{-\frac{5}{3}r^{-\frac{4}{3}} - 2r^{-\frac{7}{3}}} \quad 1$$

$$\frac{d}{dt}(\csc t + \tan t \sec t)$$

$$= -\csc t \cot t + \sec^2 t \sec t + \tan t \sec t \tan t$$

$$= \underline{-\csc t \cot t} + \underline{\sec^3 t} \; 1 + \underline{\sec t \tan^2 t} \; 1$$

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